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Reflection of Spherical Waves at Weak Interfaces

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REFLECTION OF SPHERICAL WAVES AT WEAK INTERFACES

L. Brekhovskikh

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The field of an acoustic or radiowave point source at a flat interface is investigated in the case where the properties of the media on both sides of the interface are similar. The limits of applicability of the existing theories are pointed out. Criteria are found for which transition layers may be replaced by interfaces.

Introduction

We encounter the problem of reflection of spherical waves when investigating the field of an electromagnetic or acoustic point source in the presence of an interface between two media. This problem is the subject of a large number of works in which it has been investigated basically from two points of view: a) investigation of the field at strongly reflecting interfaces, which in the case of electromagnetic waves corresponds to propagation of radiowaves over good conducting surfaces [1-4]; b) investigation of the field in the wave range with any properties of the media (5, 6, 13]. These two cases actually overlap each other. The cited papers as a group represent a complete investigation of the problem satisfying the majority of practical and experimental questions.

However, returning to the solutions obtained in these papers, let us note that they give infinitely increasing values for the amplitudes of the reflected waves on approximation of the index of refraction n to one at the interface (see part 1 in [2] and also [5, 6, 13]). Clarification of the nature of the anomaly obtained here, determination of the

limits of applicability of the existing theories and derivation of formulas which are valid for n as close as desired to one — all are of theoretical and practical interest. In particular, when investigating radio wave propagation in the atmosphere frequently it is necessary to deal with their reflection from layers which may be reduced to interfaces between media with similar dielectric constants. The index of refraction for these interfaces may differ from one by only small fractions of a % [7]. We encounter the same case on reflection of sound from layers of discontinuity in the sea. As is known, the latter are comparatively thin layers extending in the horizontal direction with large temperature and salinity gradients. The index of refraction equal to the ratio of the propagation rate of sound in the water above and below a layer of discontinuity also differs from one by only a few percent or fraction of a percent.

The interfaces of media with similar properties and, consequently, an index of refraction close to one will be called weak interfaces in the remainder of this article.

Let us note that the formulas obtained by Ryazin [12] for the Hertzian vector are also valid for weak interfaces. However, they refer only to the case where the transmitter and receiver are both at the interface.

§ 1. Limits of Applicability of the Existing Theories

Apparently we were the first to note [13] the anomaly obtained when $n \to 1$, and also the fact that investigation of the field of a reflected wave by expansion in a series with respect to powers of $1/k_0R$ (which is done by the saddle point method used in references [1, 5, 6]) is possible for all glancing angles only on fulfillment of the condition ([13], equation (28)

$$\sqrt{k_0 R |n^2-1|} \gg 1, \tag{1}$$

where R is the distance from the image radiator to the receiver. If the latter condition is not fulfilled, the saddle point method gives correct values for the fields only at sufficiently large glancing angles χ (figure 1) satisfying the condition

$$k_0 R \chi^2 \gg 1. \tag{2}$$

The inapplicability of the saddle point method for n close to one and small glancing angles may be understood from some very obvious arguments which we consider it useful to present. On reflection of a spherical wave all points of the interface do not play an essential role, but only some effective zone [14] in the shape of an ellipse the area of which

approaches zero on conversion to geometric optics where it is possible to talk about reflection of the wave from a certain point on an interface. In accord with this the field of the reflected wave at an arbitrary point in space is the result of waves going only in the directions which correspond to straight lines connecting this point with all parts of the effective zone. These will be the directions deviating by small angles on the order of $1/\sqrt{k_0 R}$ and less from the direction of the beam reflected by the laws of geometric optics. Application of the saddle point method is possible if the reflection coefficient and its derivative with respect to the angle may be considered to have low variation in this range of angles.

However, from the known expression for the reflection coefficient of plane waves reflected from the interface of two media

$$B(\chi)=\frac{m\sin\chi-\sqrt{n^2-\cos^2\chi}}{m\sin\chi+\sqrt{n^2-\cos^2\chi}} \tag{3}$$

we find that for n close to one and small χ the first and second derivatives of $B(\chi)$ may be as large as desired. Here $m = n^2$ in electrodynamics and $m = \rho_1/\rho_0$ (the density ratio of the media) in acoustics.

Application of the saddle point method gives geometric optics (acoustics) in the first approximation and the corrections to it in subsequent expressions. For n close to one, fulfillment of condition (2) ensures that the corrections will be small. Therefore, it is also a condition of applicability of geometric optics in the description of the field of a reflected wave.

§ 2. The Field of a Point Source Located at a Weak Interface

Let us investigate the field of a point source located at a height of z_0 (which may also be zero) of a flat interface at the point (r, z). In the electromagnetic case a vertical dipole will be used as the radiator, and in the acoustic case, a pulsating sphere of infinitely small radius. The field will be characterized by a scalar function φ which is the vertical component of the Hertzian vector or the acoustic potential respectively.

It may be demonstrated [9] that for any relationship of fixed media we will have

$$\varphi=\frac{e^{ik_0R_0}}{R_0}+\varphi_r, \tag{4}$$

for the field in the upper medium where the first term is the direct wave and the second is the reflected wave. Here

$$\varphi_r = \int_0^\infty \frac{mb_0 - b_1}{mb_0 + b_1} e^{-b_0\zeta} J_0(\xi r) \frac{\xi d\xi}{b_0}. \qquad (5)$$

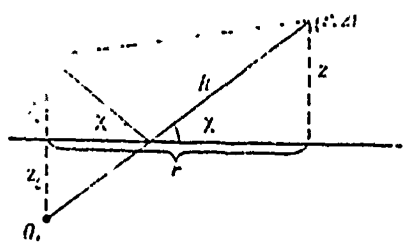

Figure 1. R_0 -- the distance from the reception point (r,z) to the radiator Q; R -- the distance from the same point to the image radiator Q_1; χ is the glancing angle formed by the path of the beam with the interface.

Here $b_0 = \sqrt{\xi^2 - k_0^2}$; $b_1 = \sqrt{\xi^2 - k_1^2}$; k_0 and k_1 are the wave numbers respectively in the upper and lower media, and m has the same value as in (3), and $\zeta = z + z_0$.

Our problem is to investigate expression (5) for $n = k_1/k_0$ close to one.

Let us expand the expression under the integral sign in (5) in a series with respect to powers of $n^2 - 1$. For this purpose we shall introduce the notation

$$x = \frac{k_1^2 - k_0^2}{b_0^2} = \frac{k_0^2(n^2 - 1)}{b_0^2}. \qquad (6)$$

Then

$$\frac{mb_0 - b_1}{mb_0 + b_1} = \frac{m - \sqrt{1-x}}{m + \sqrt{1-x}} = \sum_{s=0}^{\infty} B_s x^s. \qquad (7)$$

Substituting this expression in (5) and considering (6), for the reflected wave[1] we obtain

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[1]The radius of convergence of the series (7) is equal to one. Therefore, representation of φ_r in the form of (8) is possible only in the case where the integration path is selected in (5) on which $|x| < 1$ everywhere, that is, $|b_0|^2 > |k_0^2 - k_1^2|$. For example, we can take for this path the path which runs from $\xi = 0$ first along the real axis, then by way of the point $\xi = k_0$ along the semicircle of radius greater than $|\sqrt{k_0^2 - k_1^2}|$ lying in the fourth quadrant, and then back to the real axis. Since the expression under the integral sign in (5) has no singularities there are no obstacles to converting to this path of integration. It is assumed that in the integrals (9) obtained after expansion in the series, the integration path again coincides with the real axis from which, of course, the values of the integrals are not changed.

$$\varphi_r = \sum_{s=0}^{\infty} B_s k_0^{2s} (n^2-1)^s I_{s,r}. \tag{8}$$

where

$$I_s = \int_0^{\infty} \frac{e^{-b_0\zeta}}{b_0^{2s+1}} J_0(\xi r)\, \xi\, d\xi. \tag{9}$$

For the expansion coefficients B_s in (7) we have

$$B_0 = \frac{m-1}{m+1}, \quad B_1 = \frac{m}{(m+1)^2}, \quad B_2 = -\frac{m(m+3)}{4(m+1)^3}, \tag{10}$$

where the successive coefficients may be found by the recurrent formula

$$(m^2-1) B_s + B_{s-1} = m (B_{s-1})_{m=1}. \tag{11}$$

The integrals I_s are investigated for any s in the appendix. We shall limit ourselves here to the case of sufficiently small $n^2 - 1$ so that in (8) the second and subsequent powers of this variable may be neglected. Then, considering the known formula[1]

$$\frac{e^{ik_0 R}}{R} = \int_0^{\infty} \frac{e^{-b_0\zeta}}{b_0} J_0(\xi r)\, \xi\, d\xi, \tag{12}$$

we obtain

$$\varphi_r = \frac{m-1}{m+1} \cdot \frac{e^{ik_0 R}}{R} + \frac{m k_0^2 (n^2-1)}{(m+1)^2} I_1, \tag{13}$$

where

$$R = \sqrt{r^2 + \zeta^2}.$$

As is shown in the appendix, the integral I_1 may be represented in the form

$$I_1 = \int_0^{\zeta} \frac{e^{ik_0 R_1}}{R_1} (\zeta - t)\, dt + A + D\zeta, \tag{14}$$

where

$$R_1 = \sqrt{r^2 + t^2},$$

$$A = \frac{i}{k_0} e^{ik_0 r}, \quad D = -i \sqrt{\frac{\pi}{2k_0 r}}\, e^{i\left(k_0 r - \frac{\pi}{4}\right)}. \tag{15}$$

It remains for us to investigate the integral term in (14). If we divide it into the difference of two integrals, we have

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[1] See [3], page 941.

$$\int_0^{\zeta} \frac{e^{ik_0R_1}}{R_1}(\zeta - t)\,dt = \zeta\int_0^{\zeta} \frac{e^{ik_0R_1}}{R_1}\,dt - \int_0^{\zeta} \frac{e^{ik_0R_1}}{R_1}\,t\,dt. \tag{16}$$

Here the second integral is taken directly, and on substitution of the limits it gives

$$\frac{i}{k_0}\left(e^{ik_0R} - e^{ik_0r}\right). \tag{17}$$

To calculate the first integral let us replace the variables by the formula

$$k_0(R_1 - r) = \alpha^2, \quad \text{from which} \quad \frac{k_0 t\,dt}{R_1} = 2\alpha\,d\alpha,$$

then

$$\int_0^{\zeta} \frac{e^{ik_0R_1}}{R_1}\,dt = 2e^{ik_0r}\int_0^{k_0(R-r)} \frac{e^{i\alpha^2}\,d\alpha}{\sqrt{\alpha^2 + 2k_0r}}. \tag{18}$$

We shall assume that

$$\frac{\zeta^2}{r^2} \ll 1,$$

and then on the entire integration path $\alpha^2 \cong \frac{k_0 t^2}{r} \ll k_0 r$, as a result of which the first term under the square root in (18) may be neglected.

As a result we obtain

$$\int_0^{\zeta} \frac{e^{ik_0R_1}}{R_1}\,dt = \sqrt{\frac{\pi}{k_0 r}}\,[C(u) + iS(u)], \tag{19}$$

where

$$u = \sqrt{\frac{2k_0}{\pi}(R - r)} \cong \sqrt{\frac{k_0\zeta^2}{\pi r}}, \tag{20}$$

and $C(u)$ and $S(u)$ are the Fresnel integrals. Substituting this result in (14) and considering (19) we obtain the expression for I_1. On substituting this in the expression for the reflected wave (13) we have

$$\varphi_r = \frac{e^{ik_0R}}{R}\left\{\frac{m-1}{m+1} - \frac{im(1-n^2)k_0r}{(m+1)^2}\left[1 - \pi i u e^{-i\frac{\pi u^2}{2}}\left(C + iS - \frac{1+i}{2}\right)\right]\right\}. \tag{21}$$

In electrodynamics where $m = n^2$, the first term in the braces is k_0r times less than the second and consequently may be omitted. In acoustics we are interested only in the cases where $m - 1$ is on the order of or less than $n^2 - 1$ so that the first term may again be omitted.[1] As a result the reflected wave is written as

[1] When $m - 1$ is not small, we do not obtain a singularity when $n \to 1$.

$$\varphi_r = B\frac{e^{ik_0 R}}{R}, \tag{22}$$

where

$$B = \frac{im(n^2-1)k_0 r}{(m+1)^2}\left[1 - \pi i u e^{-\frac{i\pi u^2}{2}}\left(C + iS - \frac{1+i}{2}\right)\right], \tag{23}$$

and in electrodynamics the factor $m^2/(m+1)^2 = n^2/(n^2+1)^2$ may be replaced by 1/4.

§ 3. Discussion of the Results

Expression (22) represents an ordinary spherical wave reflected with a reflection coefficient of B. It may be shown that for $u^2 \gg 1$ it coincides with the reflection coefficient of plane waves (3) if the latter is also expanded in a series with respect to powers of $n^2 - 1$ and is limited to the first power. This must be expected inasmuch as the condition $u^2 \gg 1$ which may also be written in the form

$$\frac{k_0 \zeta^2}{R} \gg 1, \tag{24}$$

is equivalent to condition (2) on fulfillment of which geometric optics is valid.

Let us introduce the new variable

$$x = \frac{1}{2}\pi u^2$$

in place of u, and let us use the asymptotic expansion of the Fresnel integrals [8] with respect to powers of $1/x$.

Then we obtain

$$C + iS - \frac{1+i}{2} = -\frac{1}{\sqrt{2\pi x}}\left(i + \frac{1}{2x} - \frac{1\cdot 3}{(2x)^2}\, i + \ldots\right).$$

Substituting this in (23) and considering that $\sin\chi = \zeta/R$, dropping small values of

$$B_0 = \frac{m(1-n^2)}{(m+1)^2 \sin^2\chi}, \tag{25}$$

we obtain for B the same expression as we obtained on expanding (3) with respect to powers of $n^2 - 1$ limiting ourselves to the first term and dropping the term $\frac{m-1}{m+1}$ in accord with the above indicated arguments.

When $u \ll 1$ in (23) all terms in brackets, except the 1, may be neglected as a result of which the reflection coefficient B again assumes a simple form.

It is of interest to trace the value of the ratio B/B_0 for all values of u.

According to (23) and (25), we have

$$\frac{B}{B_0} = -i\pi u^2 \left[1 - \pi i u e^{-i\frac{\pi u^2}{2}} \left(C + iS - \frac{1+i}{2} \right) \right]. \tag{26}$$

The modulus of this quantity is depicted in figure 2 as a function of u where the logarithmic scale is taken along the axes. With an increase in u, $|B/B_0|$ increases continuously. This increase first proceeds according to the law u^2 which on the assumed scale corresponds to a straight line. For large u, $|B/B_0|$ approaches zero which conditions the applicability of geometric optics, that is, coincidence of the reflection coefficients of spherical and plane waves. This result refers to the case of a sufficiently thin transition layer (see § 4) when the latter may be replaced by an interface. For the case of thick transition layers we have only the calculations of reflection of plane waves.[1]

We may consider that the limits of applicability of these calculations to spherical waves are also determined by the condition (24).

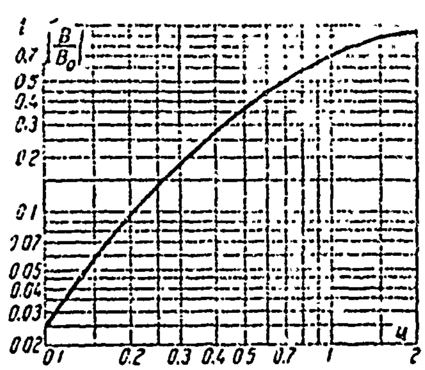

Figure 2. Modulus of the ratio B/B_0 as a function of a parameter u. B is the reflection coefficient ensuing from the exact theory of reflection of a spherical wave; B_0 is the reflection coefficient for a plane wave (geometric optics or acoustics).

Let us present some examples where condition (24) is not fulfilled. For the case of radio waves with location of the weak interface at a height of 400 meters above the earth and at distances of 50 km the value of $k_0\zeta^2/R$ is 10 when $\lambda = 2$ meters and 2 when $\lambda = 10$ meters.

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[1] For a detailed bibliography see the article by Vvedenskiy and Arenberg [7].

In the latter case the field of the reflected wave must be calculated by formulas (22) and (23) obtained above.

In hydroacoustics with a depth of the layer of discontinuity of 30 meters and at a distance of R = 2 km the reflected wave cannot be calculated by the laws of geometric acoustics for $\lambda \geq 1$ meter.

In practical respects investigation of the propagation of sound or radio waves in a layer bounded on one side by a weak interface is of interest. We demonstrated earlier [9] that in the most general case the field in the layer may be broken down into an infinite sum of waves reaching the receiver after a different number of reflections from the interface. In the investigated case the amplitude of a wave undergoing k reflections from a weak interface will have a factor of $(n^2 - 1)^k$ from which it follows that in our approximation it is necessary to consider only the following six waves not having one or only one reflection from this interface: 1) a direct wave, 2) a wave reflected from an ordinary interface, c) a wave reflected from a weak interface, 4) a wave reflected from an ordinary and then a weak interface, 5) the same with the inverse order of reflections, 6) a wave reflected twice from an ordinary interface and once from a weak interface. The second of the enumerated waves is calculated by the formulas obtained above [13], and we investigated the third one above. The fourth, fifth and sixth reduce to the third if the ordinary interface is an absolutely reflecting interface as, for example, the surface of the water in the hydroacoustic case. Here the wave amplitude will be calculated by formulas (22) and (23) where formula (20) will be valid for u if by ζ we mean the projection of the path followed by the corresponding beam to the z-axis, and $R = \sqrt{r^2 + \zeta^2}$. In addition, if for an absolutely reflecting interface the reflection coefficient B_0 = -1, as in the above indicated hydroacoustic case, when the second interface is not absolutely reflecting, the problem is somewhat more complicated, but it may also be solved with the help of expansions of the solution in a series with respect to powers of $n^2 - 1$ and subsequent application of the saddle point method. However, we shall not discuss this.

§ 4. Replacement of the Transition Layer by an Interface

In the majority of practical cases a transition layer occurs in place of a weak interface. The question arises as to the cases in which this layer may be idealized as an interface. It is known that in ordinary cases this may be found if the thickness of the layer is small by comparison with the wavelength. Weak interfaces also have a peculiarity in this case since the corresponding condition for them turns out to be weaker. To derive the latter let us use some obvious arguments.

Let the transition layer with a thickness l be parallel to the plane z = 0. Let us consider reflection of the plane wave incident on it at a glancing angle χ. We are interested in the cases where l is comparable to the wave length λ or greater than it where the difference in propagation rates on the upper and lower boundaries of the layer is small. Under these conditions the propagation of a plane wave will be

subject to the laws of geometric optics so that the dependency of the phase of the wave on the coordinate z will be given by the factor exp $[i \int k_z dz]$. The extent of the layer in the z direction may be neglected if the phase inroad in the thickness of the layer is small, That is, if

$$\int_0^l k_z\, dz \ll 1.$$

The latter condition will be reinforced by replacing k_z by its maximum value in the layer as a result of which it is written as

$$(k_z)_{max}\, l \ll 1. \tag{27}$$

With a monotonic change in k_z within the boundaries of the layer (only this case is of interest to us) the maximum value of k_z is reached on one boundary of the layer as a result of which the preceding condition is entirely equivalent to the following two conditions

$$(k_z)_0\, l \ll 1 \quad (k_z)_1 l \ll 1, \tag{28}$$

where the indices 0 and 1 refer to the media located above and below the layer respectively. Since $(k_z)_0 = 2\pi/\lambda_0 \sin\chi$ and $(k_z)_1 = 2\pi/\lambda_1 \sin\chi_1$ where χ_1 is the glancing angle of the refracted wave, condition (28) is rewritten in the form

$$l \sin\chi \ll \lambda_0/2\pi, \quad l \sin\chi_1 \ll \lambda_1/2\pi. \tag{29}$$

In the case of weak interfaces $\chi_1 \simeq \chi$, $\chi_0 \simeq \chi_1$, and then both conditions reduce to one condition

$$l \sin\chi \ll \lambda/2\pi. \tag{30}$$

For small glancing angles the last condition may be fulfilled for l comparable to and even greater than λ.

The conversion from plane waves to spherical waves causes complications at first glance. Actually, on expansion of the spherical wave in plane waves ([2], and also [3], page 943) the wave must be considered with a wave vector component along the z axis as large as desired and imaginary. For large k_z the condition (27) is not fulfilled. However, this difficulty

disappears if we consider the field in the wave zone ($k_0R >> 1$). Then as was pointed out in § 2, only plane waves will play a significant role which have glancing angles close to the angle χ depicted in figure 1. This angle must be substituted in condition (30).

Appendix

The integrals (9) must be represented in a somewhat different form. Differentiating I_s for this purpose $2s$ times with respect to ζ and considering (12), we obtain

$$\frac{d^{2s} I^s}{d\zeta^{2s}} = \frac{e^{ik_0 R}}{R},$$

where

$$R = \sqrt{r^2 + \zeta^2}.$$

As is easy to check by differentiation, the last equation is satisfied by the function

$$I_s = \frac{1}{(2s-1)!} \int_0^{\zeta} \frac{e^{ik_0 R_1}}{R_1} (\zeta - t)^{2s-1} dt + c_0^{(s)} + c_1^{(s)} \zeta + \ldots + c_{2s-1}^{(s)} \zeta^{2s-1},$$

where $R_1 = \sqrt{r^2 + t^2}$.

For $c_0^{(s)}$, $c_1^{(s)}$, ..., by sequentially differentiating the last expression and assuming $\zeta = 0$ we obtain

$$c_0^{(s)} = (I_s)_{\zeta=0}, \quad c_1^{(s)} = \left(\frac{dI_s}{d\zeta}\right)_{\zeta=0}$$

and in general

$$c_l^{(s)} = \frac{1}{l!} \left(\frac{d^l I_s}{d\zeta^l}\right)_{\zeta=0} = \frac{(-1)^l}{l!} \int_0^{\infty} \frac{J_0(\xi r)\, \xi\, d\xi}{b_0^{2s-l+1}}.$$

Using the formulas given by Watson [10] for integrals of this type (see also the paper by Fock [11]), we obtain

$$c_l^{(s)} = \frac{\pi}{2} \left(\frac{r}{2k_0}\right)^{s - \frac{l+1}{2}} \frac{(-1)^s e^{i\frac{\pi l}{2}} H_{s-\frac{l-1}{2}}(k_0 r)}{l!\, \Gamma\left(s - \frac{l-1}{2}\right)}.$$

The variables A and D used in the text are equal respectively to $c_0^{(1)}$ and $c_1^{(1)}$. By replacing the Hankel functions by their asymptotic representations, from the last formula we obtain the values of (15) for them.

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13. ABSTRACT

Reflection of both acoustic and electromagnetic spherical waves at a weak interface is considered. The limits of applicability of existing theories are established, and reflection formulas are derived for indexes of refraction close to one.

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